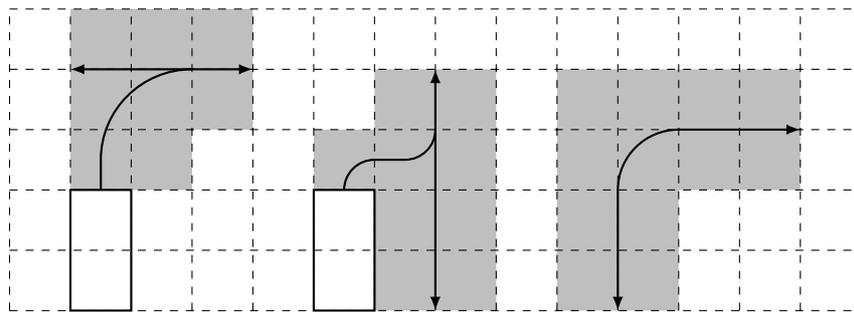


## SIMC 2020 Online Challenge: Car Park Efficiency

The SIMC Online Challenge for 2020 concerns the efficient design of a car park (parking lot). In many cities worldwide, space is at a premium, and with the increasing number of cars it is important that car parks utilise their space as best they can, to allow as many cars to park as possible. This challenge involves both the layout in an ‘idealised’ model and also what happens when we consider a more realistic model of how cars actually turn.

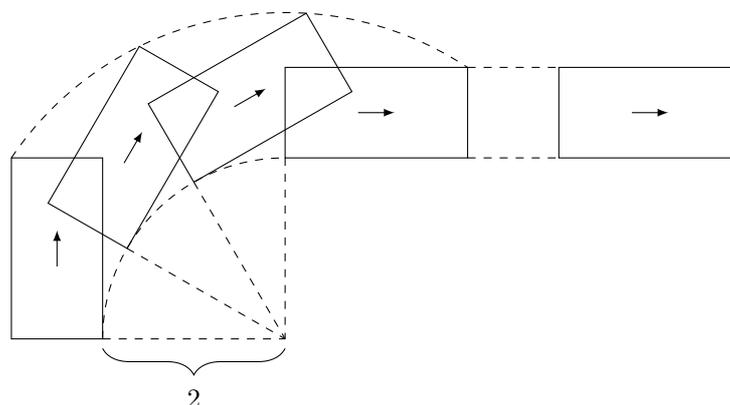
The efficiency of a car park is defined as the total area of its parking spaces divided by its total area (used or unused). The car park can be modelled on a grid of unit squares according to the following rules, which are illustrated below. Each parking space is a  $1 \times 2$  rectangle. If the space is accessed from the end via a turn, then a  $3 \times 3$  square in front of it needs to be empty (except for a corner). If the space is accessed from the side, then it must form part of an otherwise empty  $3 \times 4$  square (except for a corner). The aisles of the car park, which cars use to travel from the entrance to each space, must be at least 2 squares wide. The boundaries of the car park cannot be crossed except for at a single entrance (and a single exit, if necessary).



**Question 1.** Determine the approximate maximum efficiency of a car park built on a square area of land of size  $N \times N$ , where  $N$  is very large.

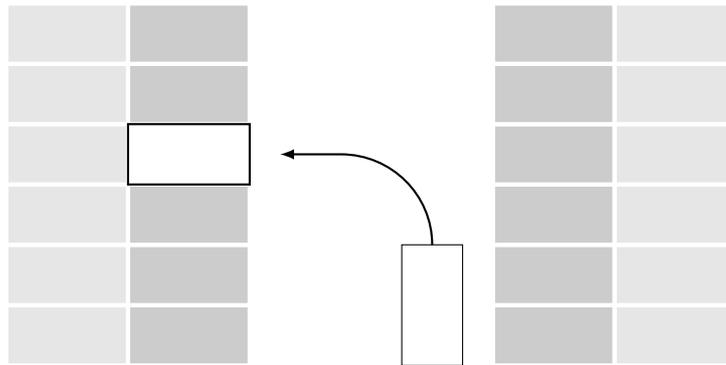
**Question 2.** Determine the approximate maximum efficiency of a car park built on a rectangular area of land of size (a)  $6 \times N$  and (b)  $10 \times N$ , again where  $N$  is very large.

We now consider the manoeuvring in more detail. The car can be modelled as a  $1 \times 2$  rectangle that slides forwards/backwards as well as rotates around a point that lies on the line passing through its back bumper at a distance 2 from the car, as shown below. The car must be able to manoeuvre into each parking space without any part of it encroaching into any other space.

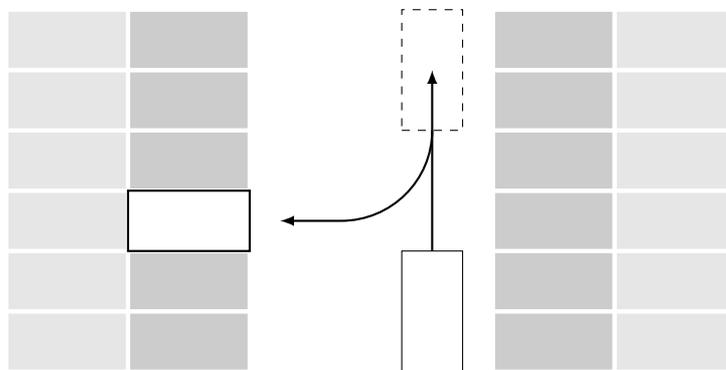


**Question 3.** Suppose that the car park is very large in both directions and consists of parallel aisles with spaces perpendicular to the aisles. Since the cars are the same size as the spaces, each car has to be perfectly aligned with the space before entering it from the aisle.

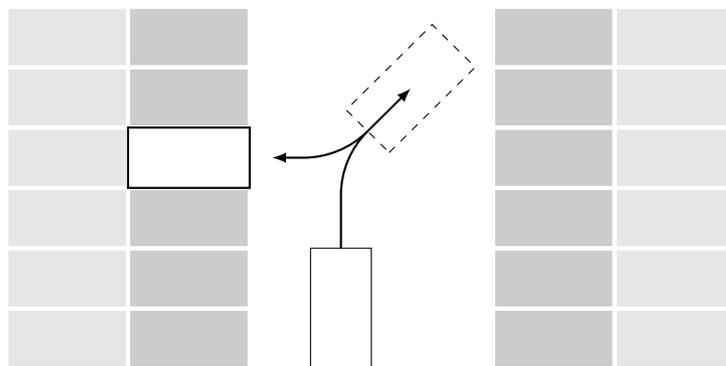
(a) What is the approximate maximum efficiency, if the aisles are wide enough that a car can turn directly into each space?



(b) What if the car can instead drive straight past the space and then reverse into it?



(c) What if the car can turn while driving past the space before reversing into it?



**Question 4.** For this detailed manoeuvring model, how else could the car park be laid out, and what approximate maximum efficiency do you obtain?