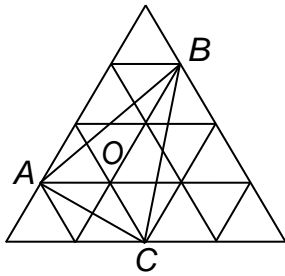


NMOS SPECIAL ROUND 2018 SOLUTION

1



$$\begin{aligned} \text{Area of } \triangle AOB &= \text{Area of } \triangle BOC \\ &= \frac{1}{2} \times (\text{areas of 4 small triangles}) = 10 \text{ cm}^2 \end{aligned}$$

$$\text{Area of } \triangle AOC = \frac{1}{2} \times (\text{areas of 2 small triangles}) = 5 \text{ cm}^2$$

Hence, area of $\triangle ABC$ is 25 cm^2 .

- 2 Suppose currently the elder brother is m years old and the younger brother is n years old. Hence, a few years ago, the elder brother was n years old and the younger brother was $\frac{m}{2}$ years old.

$$m - n = n - \frac{m}{2} \Rightarrow \frac{3m}{2} = 2n \Rightarrow m : n = 4 : 3.$$

Thus, the elder brother is currently $\frac{42}{7} \times 4 = 24$.

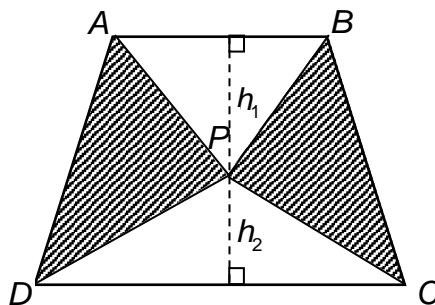
- 3 By division, 1082 has remainder 2 when divided by 12.

The smallest 3-digit number starting with 2 that is a multiple of 12 is $12 \times 17 = 204$.

The largest 3-digit number starting with 2 that is a multiple of 12 is $12 \times 24 = 288$.

Hence, the last 2-digit number \overline{ab} could take 8 different values.

4



Since $\frac{AB}{CD} = \frac{2}{3}$, let $AB = 2a$ and $CD = 3a$.

$$\text{Area of } \triangle APB = 10 = \frac{1}{2} \times (2a) \times h_1, \text{ hence, } h_1 = \frac{10}{a}.$$

$$\text{Area of } \triangle CPD = 12 = \frac{1}{2} \times (3a) \times h_2, \text{ hence, } h_2 = \frac{8}{a}.$$

Area of trapezium $ABCD = \frac{1}{2} \times (2a + 3a) \times \left(\frac{10}{a} + \frac{8}{a} \right) = 45 \text{ cm}^2$. Thus, the area of the shaded region is $45 - 10 - 12 = 23 \text{ cm}^2$.

- 5 Define $A = \{\text{numbers are multiples of } 5\}$,
 $B = \{\text{numbers have remainder } 1 \text{ when divided by } 5\}$,
 $C = \{\text{numbers have remainder } 2 \text{ when divided by } 5\}$,
 $D = \{\text{numbers have remainder } 3 \text{ when divided by } 5\}$,
 $E = \{\text{numbers have remainder } 4 \text{ when divided by } 5\}$.

To make the sum a multiple of 5, there are three cases.

Case 1. Both numbers are chosen from set A .

$$\text{There are } \frac{20 \times 19}{2} = 190 \text{ different pairs.}$$

Case 2. One number is chosen from set B , and the other is chosen from set E .

$$\text{There are } 20 \times 20 = 400 \text{ different pairs.}$$

Case 3. One number is chosen from set C , and the other is chosen from set D .

$$\text{There are } 20 \times 20 = 400 \text{ different pairs.}$$

Hence, there are altogether $190 + 400 + 400 = 990$ different pairs.

6 By the definition of $\{x\}$,

$$\left\{\frac{2018+1}{5}\right\} = \frac{4}{5}, \left\{\frac{2018+2}{5}\right\} = 0, \left\{\frac{2018+3}{5}\right\} = \frac{1}{5}, \left\{\frac{2018+4}{5}\right\} = \frac{2}{5},$$
$$\left\{\frac{2018+5}{5}\right\} = \frac{3}{5}, \dots, \left\{\frac{2018+2017}{5}\right\} = 0, \left\{\frac{2018+2018}{5}\right\} = \frac{1}{5}.$$

Hence,

$$\left\{\frac{2018+1}{5}\right\} + \left\{\frac{2018+2}{5}\right\} + \left\{\frac{2018+3}{5}\right\} + \dots + \left\{\frac{2018+2017}{5}\right\} + \left\{\frac{2018+2018}{5}\right\}$$
$$= \left(\frac{1+2+3+4+0}{5}\right) \times \frac{2015}{5} + \frac{4}{5} + \frac{1}{5} = 807.$$

7 To get the largest possible number of pupils who passed all three tests, the 43 pupils would pass exactly one test or all three tests.

Hence, at most $\frac{30+28+25-43}{2} = 20$ pupils passed all three tests.

8 Let the four distinct positive whole numbers be a, b, c and d in descending order.

According to the question,

$$\frac{1}{3}(a+b+c)+d=39 \quad \text{①}$$

$$a+\frac{1}{3}(b+c+d)=51 \quad \text{②}$$

$$\text{②} - \text{①}: \frac{2}{3}a - \frac{2}{3}d = 12. \text{ Hence, } a = d + 18.$$

Since b and c are at least $d+2$ and $d+1$ respectively,

$$\frac{1}{3}(18+d+d+1+d+2)+d \leq 39$$

Hence, $d \leq 16$, and $a = 18 + d \leq 34$. In fact, 34, 18, 17 and 16 could be the four whole numbers.

9 Since $BD = 3BE$, Area of $\triangle ABD = 3 \times$ (Area of $\triangle ABE$).

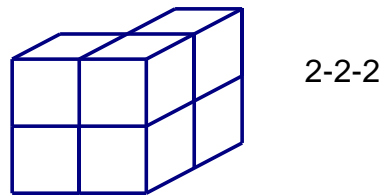
Since $CF = GF$, Area of $\triangle CFD =$ Area of $\triangle GFD$.

Since $BD = 3FD$, Area of $\triangle BCD = 3 \times$ (Area of $\triangle CFD$) = $3 \times$ (Area of $\triangle GFD$). Hence,

$$\begin{aligned} \text{Area of } ABCD &= \text{Area of } \triangle ABD + \text{Area of } \triangle BCD \\ &= 3 \times (\text{Area of } \triangle ABE + \text{Area of } \triangle GFD) = 39 \text{ cm}^2. \end{aligned}$$

10 Clearly, the bottom layer must be of the corner shape. Now we are to distribute the other three cubes into the 2nd or higher layers.

Case 1. If all the three cubes are on the 2nd layer, there is only one structure as shown below. We will denote it as '2-2-2' (number of cubes on the left – in centre – on the right).

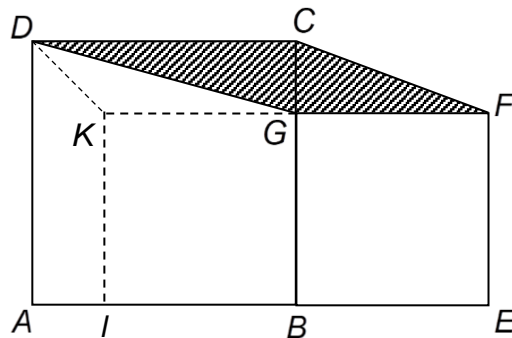


Case 2. If all the three cubes are piled up, there are 3 possible structures: 4-1-1, 1-4-1, and 1-1-4.

Case 3. If two of the three cubes are on the 2nd layer and the remaining cube is on the 3rd layer, there are 6 possible structures: 3-2-1, 3-1-2, 2-3-1, 2-1-3, 1-2-3, and 1-3-2.

Hence, a total of 10 possible structures could be built.

11



Inside the square $ABCD$, draw square $BGKI$ which is of same area as square $BEFG$. Hence, area of $AIKGCD$ is 74 cm^2 .

Quadrilaterals $KDAI$ and $KDCG$ are of the same area, hence,

$$\text{Area of quadrilateral } KDCG = \frac{1}{2} \times 74 = 37 \text{ cm}^2.$$

Since triangles KGD and CGF are of the same area, the area of quadrilateral $CDGF$ is 37 cm^2 .

- 12 To pull up the average score to 95 marks, Paul needs at least $(95 - 89) \times 4 = 24$ marks. If he is able to score full marks for the subsequent tests, $100 - 95 = 5$ marks from each test will contribute to pull up the average score.

Since $24 \div 5 = 4 \text{ R } 4$, Paul will take at least 5 more tests.

- 13 The largest possible 3-digit number which is a multiple of 75 is $13 \times 75 = 975$, and the other multiple of 75 which has two equal digits with 975 is $10 \times 75 = 750$.

Hence, the largest possible sum is $975 + 750 = 1725$.

- 14 Since Jaslin gets twice as many marbles as George, the total number of marbles that Jaslin and George have should be a multiple of 3. By trial and error, the only possible case is that Jaslin takes three bags of 19, 37 and 40 marbles respectively, and George takes two bags of 21 and 27 marbles respectively. Hence, the remaining bag of 32 marbles is the bag of red marbles only.

- 15 Before Peter and Queenie met each other,

	Peter	Queenie
Speed	3	2
Distance cycled	3	2

Suppose Peter cycled 3 units of distance and Queenie cycled 2 units before they met each other.

After they met,

	Peter	Queenie
Speed	$3 \times (1 + 20\%)$	$2 \times \left(1 + \frac{1}{3}\right)$
	27	20
Distance cycled	27	20

Suppose Peter cycled 27 parts of distance and Queenie cycled 20 parts during the period from the time they met until Peter reached place *B*.

Peter cycled 3 units + 27 parts, which is the total distance from *A* to *B*. While Queenie cycled 2 units + 20 parts.

Hence, 1 unit + 7 parts = 41 km, and 3 units + 27 parts = 3 units + 2 units.

By solving the equations, 1 unit = 27 km.

Thus, the total distance between place *A* and *B* is 135 km.

- 16 Clearly, at least one digit will appear 10 times or more. Call it digit A .

Now every digit except for A corresponds to not more than 4 unpainted squares (because 5 or more squares filled up with that digit will make the squares painted). Hence, at most $9 \times 4 = 36$ squares are unpainted.

We conclude that at least 64 squares are red, which is attainable when 64 squares are filled up with 0 and 4 squares are filled up with 1,2,...,9 each.

- 17 To complete the same task, the number of working computers should be inversely proportional to the time taken (in hours).

Suppose X supercomputers need to take N hours to solve the maze.

If two more supercomputers join the mission,

$$\begin{array}{c|c} \text{Time taken} & 8 : 7 \\ \hline X : (X + 2) & 7 : 8 \end{array} \quad \Rightarrow \quad X = 14.$$

If two supercomputers are not working,

$$\begin{array}{c|c} X : (X - 2) & 14 : 12 \\ \hline \text{Time taken} & 6 : 7 \end{array}$$

14 computers take $6 \times \frac{2}{3} = 4$ hours to solve the maze.

Hence, one supercomputer would take $14 \times 4 = 56$ hours to solve the maze.

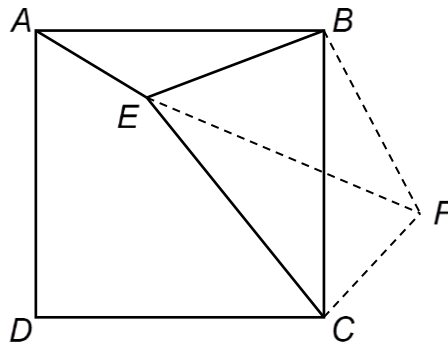
- 18 Notice that for any number m , $m * 4 = \left(\frac{4m + 16}{m + 4} \right)^2 = 16$.

$$\text{Hence, } (((1 * 2) * 3) * 4) * 5 = 16 * 5 = \left(\frac{32}{7} \right)^2.$$

Since $7^2 \times 20 = 980 < 32^2 = 1024 < 1029 = 7^2 \times 21$, the largest integer not

exceeding $\left(\frac{32}{7} \right)^2$ is 20.

- 19 Rotate $\triangle AEB$ to $\triangle CFB$ such that $BE = BF$ and $AE = CF$. Connect EF .



Let $AE = a$, $BE = 2a$, and $CE = 3a$.

$\triangle EBF$ is a right-angled isosceles triangle, hence, $\angle BFE = 45^\circ$, and

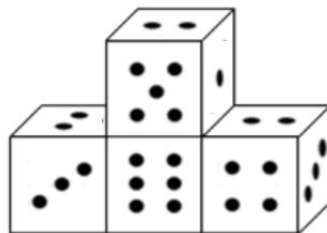
$$EF = \sqrt{8}a.$$

Since $EF^2 + CF^2 = EC^2$, $\triangle EFC$ is a right-angled triangle with $\angle EFC = 90^\circ$.

Hence, $\angle AEB = \angle CFB = 90^\circ + 45^\circ = 135^\circ$.

- 20 (Erratum corrected)

4 identical dice are glued together and form an inverted T-shape, as shown below. Each dice has 1 dots, 2 dots, 3 dots, 4 dots, 5 dots and 6 dots printed (not necessarily in the order that we are familiar with) on its 6 faces respectively. Find the largest possible total number of dots on the surface of this structure (including the base).



Solution:

Notice that 1, 3, 4 and 5 appear beside 2. Hence, $(2,6)$ must be opposite.

Since 2 faces 4 and 5 in the same orientation, $(4,5)$ must be opposite to each other. It follows that $(1,3)$ is another opposite pair.

Top: 6 dots. Front and back: $(3+1) + (6+2) + (4+5) + (5+4) = 30$ dots.

Right side: 4 dots. Left side: $3 + 4 = 7$ dots.

Bottom: at most 5 for the central dice, hence, not more than $6 + 6 + 5 = 17$ dots.

Hence, there are at most $6 + 30 + 4 + 7 + 17 = 64$ dots in total.