1. Since Benjamin had 30% more beads than Chloe, the ratio of the beads they have is $B:C=13:10$.

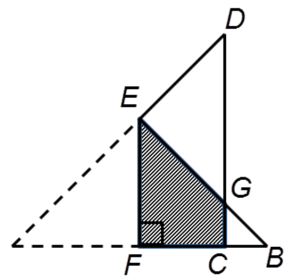
Since Benjamin had 50% fewer beads than Annie, the ratio of the beads they have is $B:A=1:2$.
Hence, the ratio of the beads is $A:B:C=26:13:10$.

	Annie	Benjamin	Chloe
Before	$26u$	$13u$	$10u$
After	$26u-90$	$13u-95$	$10u+90+95=10u+185$

From the condition given, $3(13u-95)=26u-90 \Rightarrow u=15$.
Thus, Chloe had 150 beads at first.

2. Since triangle EFB is a right-angled isosceles triangle, $EF=FB=12$ cm.

Note that $FC+FB=CD=20$ cm, hence $FC=20-12=8$ cm and $CG=CB=12-8=4$ cm.



Finally, area of the shaded region $=\frac{1}{2}\times(4+12)\times 8=64$ cm².

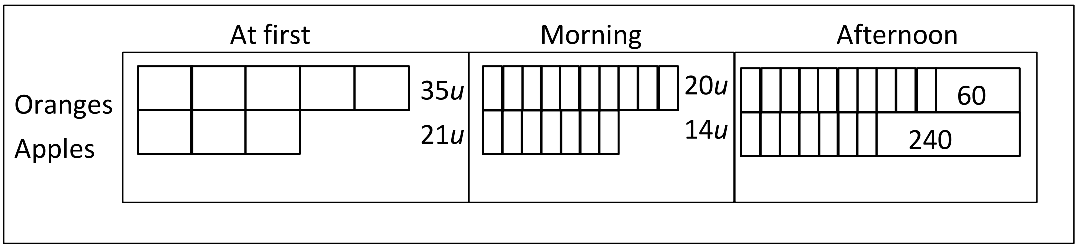
3. The number of 1×1 squares is 28; the number of 2×2 squares is 17; the number of 3×3 squares is 8 and finally the number of 4×4 squares is 2. This means that there are $28+17+8+2=55$ squares.

4. We know that $\angle GAB=\frac{5(180°)}{7}$ and $\angle RAB=\frac{3(180°)}{5}$, then

$\angle GAR=\angle GAB-\angle RAB=\frac{5(180°)}{7}-\frac{3(180°)}{5}=\frac{144°}{7}$.

Now $\angle AGR=\frac{180°-\frac{144°}{7}}{2}=\frac{558°}{7}=\frac{x°}{7}$. This implies that $x=558$.

5.



Note that $\frac{5}{3} = \frac{35}{21}$. We may suppose that there were 35*u* oranges and 20*u* apples at first. From the information, there were $35u \times \frac{4}{7} = 20u$ oranges and 14*u* apples in the morning. Finally, there were $20u + 60$ oranges and $14u + 240$ apples in the afternoon. Since the ratio of the fruits was 1:1 in the afternoon, we have $20u + 60 = 14u + 240$, which implies that $u = 30$. Hence there were $21u = 630$ apples at first.

6. Let the last digit of $1 + 2 + 3 + 4 + \cdots + n$ be *d*. The values of *d* are shown as follows:

n	1	2	3	4	5	6	7	8	9	10
d	1	**3**	6	0	5	1	8	6	5	5

n	11	12	13	14	15	16	17	18	19	20
d	6	8	1	5	0	6	**3**	1	0	0

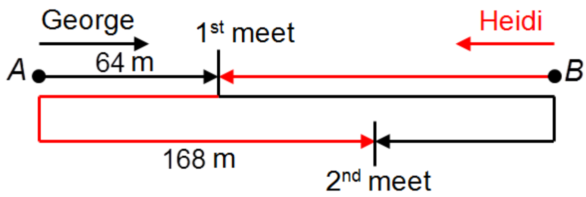
Notice that the possible values of *d* repeat after every 20 numbers. Hence the last digit of $1 + 2 + 3 + 4 + \cdots + n$ is 3 when $n = 2, 17, 22, 37, \cdots$.

Note that $1 + 2 = 3$, $1 + 2 + \cdots + 17 = \frac{17 \times 18}{2} = 153$,

$1 + 2 + \cdots + 22 = \frac{22 \times 23}{2} = 253$ and $1 + 2 + \cdots + 37 = \frac{37 \times 38}{2} = 703$.

Hence, the smallest possible value of n is 37.

7.



From the 1st meeting point to the 2nd meeting point, Heidi has travelled $64+168=232$ m.

	Distance travelled by George	Distance travelled by Heidi	Total distance travelled by both
Period I: Start $\rightarrow$ 1st meet	64 m		Distance of route *AB*.
Period II: 1st meet $\rightarrow$ 2nd meet		232 m	Double the distance of route *AB*.

$\times 2$

Due to uniform speed, the ratio of travelling distance for Heidi in Period I and II should also be $1:2$. Hence, Heidi travelled $\frac{232}{2}=116$ m when she first met George.

The total distance of route *AB* is $64+116=180$ m.

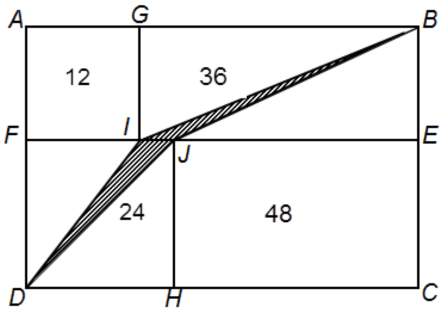
8. During sunny days, team A and B would complete $\frac{1}{12}$ and $\frac{1}{15}$ of the project respectively. Team A would complete $\frac{1}{12}-\frac{1}{15}=\frac{1}{60}$ more of the job.

During rainy days, team A would complete $\frac{1}{12}\times\left(1-40\%\right)=\frac{1}{20}$ of the project per day, and team B would complete $\frac{1}{15}\times\left(1-10\%\right)=\frac{3}{50}$ of the project per day. Team B would complete $\frac{3}{50}-\frac{1}{20}=\frac{1}{100}$ more of the job.

Suppose altogether there are m sunny days and n rainy days, since both teams complete the project on the same day, $\frac{1}{60}\times m=\frac{1}{100}\times n$. This means that $m:n=\frac{1}{100}:\frac{1}{60}=3:5$.

If $m=3, n=5$, team A would only complete $\frac{1}{12}\times 3+\frac{1}{20}\times 5=\frac{1}{2}$ of the project. Therefore, there are 10 rainy days altogether.

9. Refer to the diagram below. Let $x = FI$, $y = IJ$ and $z = JE$.



As the ratio of the bases of rectangles is the same as the ratio of the areas of rectangles, we have

$$\begin{cases} x:(y+z) = 12:36 = 1:3 = 3:9 \\ (x+y):z = 24:48 = 1:2 = 4:8 \end{cases}.$$

This means that $x:y:z = 3:1:8$.

Note that the area of triangle *IJB*: the area of rectangle *AGFI* is 1:6, and so the area of triangle *IJB* is $\frac{1}{6}(12) = 2$ cm^2.

Next, the area of triangle *IJD*: the area of rectangle *DHJF* is 1:8, and so the area of triangle *IJD* is $\frac{1}{8}(24) = 3$ cm^2.

Finally, the area of the shaded region is $2+3=5$ cm^2.

10. We claim that $M = 963$ and one possible arrangement is as follows.

$$\begin{array}{r} 4\ 1 \\ 5\ 2 \\ +\ 8\ 7\ 0 \\ \hline 9\ 6\ 3 \\ \hline \end{array}$$

In order to obtain *M*, we have $H = 9$.

$$\begin{array}{r} \boldsymbol{A}\ \boldsymbol{B} \\ \boldsymbol{C}\ \boldsymbol{D} \\ +\ \boldsymbol{E}\ \boldsymbol{F}\ \boldsymbol{G} \\ \hline 9\ \boldsymbol{I}\ \boldsymbol{J} \\ \hline \end{array} \qquad \begin{array}{r} \boldsymbol{A}\ \boldsymbol{B} \\ \boldsymbol{C}\ \boldsymbol{D} \\ +\ 8\ \boldsymbol{F}\ \boldsymbol{G} \\ \hline 9\ \boldsymbol{I}\ \boldsymbol{J} \\ \hline \end{array} \qquad \begin{array}{r} \boldsymbol{A}\ \boldsymbol{B} \\ \boldsymbol{C}\ \boldsymbol{D} \\ +\ 8\ \boldsymbol{F}\ \boldsymbol{G} \\ \hline 9\ 6\ \boldsymbol{J} \\ \hline \end{array}$$

Then $E = 8$ or $E = 7$.

Suppose $E = 7$. Then $\overline{AB} + \overline{CD} + \overline{FG} \le 84 + 63 + 52 < 200$ which means that $\overline{AB} + \overline{CD} + \overline{EFG} = 700 + \left(\overline{AB} + \overline{CD} + \overline{FG}\right) < 900$, a contradiction.

Now $E = 8$.

The largest number left is 7.

Suppose $I = 7$. Then $\overline{AB} + \overline{CD} + \overline{FG} \le 63 + 52 + 41 < 170$ which means that $\overline{AB} + \overline{CD} + \overline{EFG} = 800 + \left(\overline{AB} + \overline{CD} + \overline{FG}\right) < 970$, a contradiction.

Now $I = 6$ and as $\overline{HIJ} = 963$ is possible, we need to show that $\overline{HIJ} \ne 967$, 965 or 964.

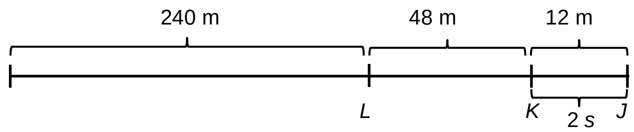
Since $A + B + C + D + F + G + J = 0 + 1 + 2 + 3 + 4 + 5 + 7 = 22$ and, $\overline{AB} + \overline{CD} + \overline{FG} = 10\left(A + C + F\right) + \left(B + D + G\right) = 9\left(A + C + F\right) + \left(22 - J\right)$ and

$\overline{AB} + \overline{CD} + \overline{FG} = 160 + J$, this means that $9\left(A + C + F\right) = 138 + 2J$.

Note that $138 + 2J$ is not a multiple of 9 if $J = 7$, 5 or 4.

This completes the proof.

11.



Suppose Justin spent *t* seconds to finish the race. Then Justin's speed is $\frac{300}{t}$ m s^{-1}. This implies that Kaden's and Leon's speeds are $\frac{288}{t}$ m s^{-1} and $\frac{240}{t}$ m s^{-1} respectively. From the condition, it is known that Kaden's speed is $\frac{12}{2}=6$ m s^{-1} and hence $\frac{288}{t}=6 \Rightarrow t=48$ s.

Now Leon's speed is $\frac{240}{48}=5$ m s^{-1} and he has to run for another $\frac{60}{5}-2=10$ s when Kaden finishes the race.

12. Since $\angle AOB=\angle BOC=\angle COD=\angle DOE$, every time Adrian moved to the next point on the circle, he rotated $\frac{360^\circ-36^\circ}{4}=81^\circ$ relative to centre O. Before Adrian reached point A for the second time, the total angle that rotated relative to centre O should be a common multiple of 81° and 360°. The least common multiple of 81 and 360 is 40×81. Hence, from the point F, Adrian reached at least $40-5=35$ points before he got to the point A for the second time.

13. Let *a*, *b*, *c*, *d* and *e* be the 5 whole numbers such that $a<b<c<d<e$. Now we have $\frac{a+b+c+d}{4}=36$, $\frac{a+b+c+e}{4}=38$, $\frac{a+b+d+e}{4}=39$, $\frac{a+c+d+e}{4}=45$ and $\frac{b+c+d+e}{4}=49$. Take the sum of these equations, we obtain $a+b+c+d+e=36+38+39+45+49=207$. Now the largest whole number among these 5 numbers is *e,* which is $(a+b+c+d+e)-(a+b+c+d)=207-4(36)=63$.

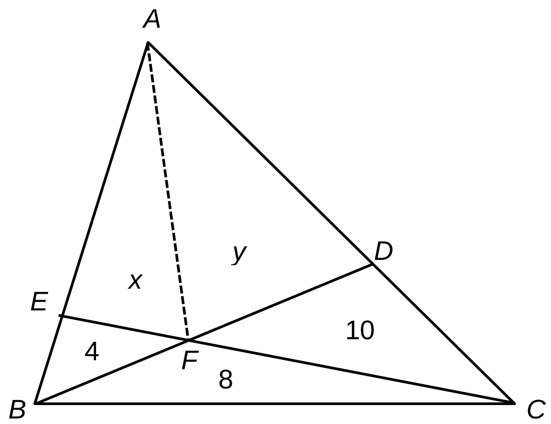
14. In every minute, the minute hand moves $\frac{1}{60}\times 360^\circ = 6^\circ$ while the hour hand moves $\frac{1}{60}\times\frac{360^\circ}{12} = 0.5^\circ$. At 11 o'clock, the angle between the minute hand and the hour hand is 30°. Then x minutes after 11 o'clock, the angle between the minute hand and the hour hand is $30^\circ + 5.5x^\circ$.

Suppose $x = x_1$ and $x = x_2$ are when the hands make an angle of 70 degrees. Then $\begin{cases} 30^\circ + 5.5x_1{}^\circ = 70^\circ \\ 30^\circ + 5.5x_2{}^\circ = 290^\circ \end{cases}$. The difference of the two equations give $5.5(x_2 - x_1) = 220$, which means that the time difference is $x_2 - x_1 = \frac{220}{5.5} = 40$.

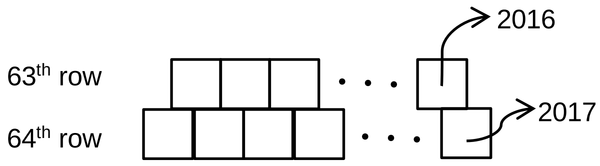
15. Join the points A and F by a line segment. Let the area of triangle AFE be x cm^2 and the area of triangle AFD be y cm^2. The area of the quadrilateral $AEFD$ is $(x+y)$ cm^2. For two triangles with the same height, we know that the ratio of their areas is equal to the ratio of their bases.

Thus $\frac{4+x}{y} = \frac{8}{10} = \frac{4}{5}$ and $\frac{x}{y+10} = \frac{4}{8} = \frac{1}{2}$. From the second equation, $y = 2x - 10$, and from the first equation, $20 + 5x = 4y = 4(2x - 10)$, which means that $x = \frac{60}{3} = 20$ and $y = 2x - 10 = 2(20) - 10 = 30$.

Now $x + y = 50$.



16. If k is odd, then the number placed in the k^{th} row, k^{th} entry is $\frac{k(k+1)}{2}$. Note that if $k=63$, the value of $\frac{k(k+1)}{2}$ is $\frac{63(64)}{2}=2016$.



Therefore 2017 is placed at 64^{th} row and 64^{th} entry. So $M=N=64$ and thus $M+N=128$.

17. As $\frac{3}{10}<\frac{r}{s}<\frac{5}{16}$, we have $\frac{16r}{5}<s<\frac{10r}{3}$.

If $r=1$, then $3.2=\frac{16}{5}<s<\frac{10}{3}=3.33$, so s is not a whole number.

If $r=2$, then $6.4=\frac{32}{5}<s<\frac{20}{3}=6.67$, so s is not a whole number.

If $r=3$, then $9.6=\frac{48}{5}<s<\frac{30}{3}=10$, so s is not a whole number.

If $r=4$, then $12.8=\frac{64}{5}<s<\frac{40}{3}=13.33$, so $s=13$.

18. Suppose $13 \times N = \overline{abc2017}$. Then the last digit of N is 9.

Now let the second last digit of N be p. Note that $13 \times 9 = 117$. This means $13p$ ends in 0. So p is 0 (see Figure (a))

Next, let the third last digit of N be q. This means $13q$ ends in 9. So q is 3. (see Figure (b))

Next, let the 4^{th} last digit of N be r. Note that $13 \times 3 = 39$. This means $13r$ ends in 8. So r is 6. (see Figure (c))

Finally, let the 5^{th} last digit of N be s. Note that $13 \times 6 = 78$. This means $13s + 7 > 100$. So s is 8. (see Figure (d))

Now the smallest value of $\overline{abc}$ is 112. (see Figure (d))

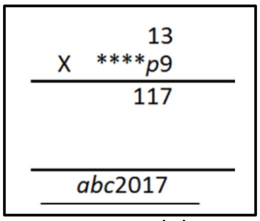


Figure (a)

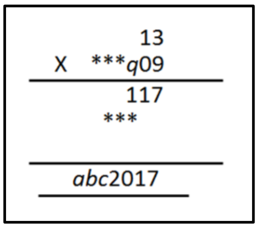


Figure (b)

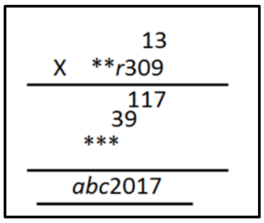


Figure (c)

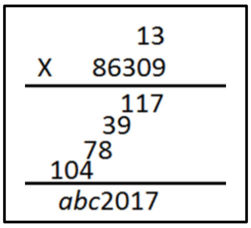


Figure (d)

19. If statement *A* is true, then the sum of the digits of the block number should be a multiple of 9 and so statement *C* must be false. Hence statement *A* and *C* does not hold at the same time.

Since $89100 = 2^2 \times 3^4 \times 5^2 \times 11$, if the number is a factor of 89100, it should not be a multiple of 7. Hence statement *B* and *E* does not hold at the same time.

This implies that statement *D* is true. Hence the 3-digit number is k^2 for some $k = 11, 12, \cdots, 31$.

Now we are left with 4 possible cases:

(i) Statements *B*, *A* and *D* are true.

(ii) Statements *B*, *C* and *D* are true.

(iii) Statements *E*, *C* and *D* are true.

(iv) Statements *E*, *A* and *D* are true.

If statement *B* is true, then the 3-digit number is $(7m)^2$ for some $m = 2, 3, 4$, which means that $14^2 = 196$, $21^2 = 441$ and $28^2 = 784$ are the possible 3-digit number. None of these make statements *A* or *C* true.

This means that statement *E* is true. We are left with 2 possible cases:

(iii) Statements *E*, *C* and *D* are true.

(iv) Statements *E*, *A* and *D* are true.

Since statements *E* and *D* are true, the 3-digit number is $\left(2^r \times 5^s \times 3^t\right)^2$ for some $r, s = 0, 1$ and some $t = 0, 1, 2$.

If the sum of the digits of the 3-digit number is not a multiple of 9, then the 3-digit number does not contain a factor of 9, which means that it must be $(2 \times 5)^2 = 100$, which means the sum of the digits is 1. Hence statement *C* is not true.

Finally, statements *E*, *A* and *D* are true and the 3-digit number is $2^2 \times 3^4 = 324$.

20. The sum is 52.

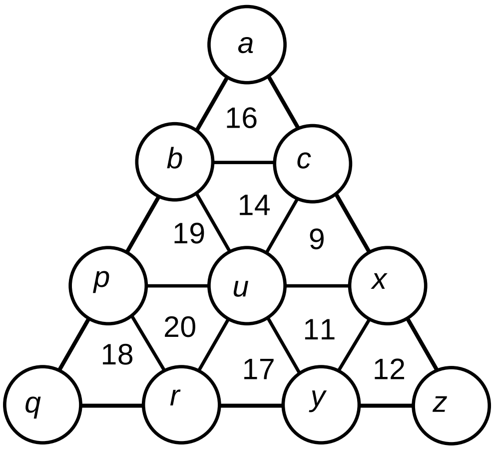


Figure (A)

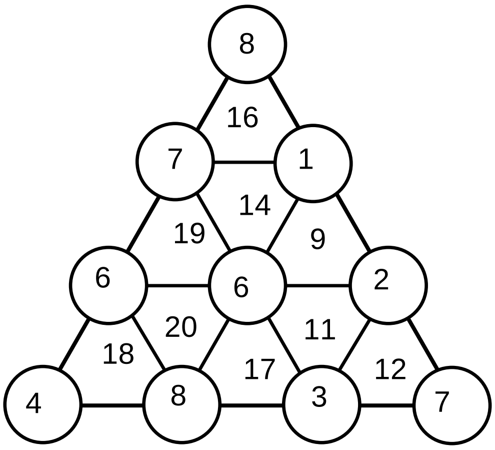


Figure (B)

Refer to the Figure (A) above, the sum is

$$(a+b+c)+(p+q+r)+(x+y+z)+u\,.$$

We know that $a+b+c=16$, $p+q+r=18$ and $x+y+z=12$. Hence the sum is $16+18+12+u=46+u$.

Note that $a+b+c=16$ and $u+b+c=14$. The difference of the equations gives $u=a-2\le 8-2=6$. Now the desired sum is not more than 52.

The Figure (B) above shows a possible solution when the sum is 52. So the largest possible sum is 52.